# The Chemical Abstract Machine (CHAM)

- Berry and Boudol (1992)
- Both a model of concurrency and a specific semantic style
- Chemical metaphor
  - States regarded as chemical *solutions* containing floating *molecules*
  - Molecules can interact with each other by means of *reactions*
  - Reactions take place *concurrently*, *unrestricted by context*
  - Solutions are encapsulated within new molecules, using *membranes*
    - The following is a solution containing *k* molecules:

$$\{\!|\, m_1 \;\; m_2 \;\; \ldots \;\; m_k \,|\!\}$$

# CHAM Syntax and Rules

Airlock

$$
\begin{aligned}
Molecule &::= Solution \mid Molecule \triangleleft Solution \\
Solution &::= \{\!|\, \mathbf{Bag}\,\{Molecule\} \,|\!\}
\end{aligned}
$$

# CHAM Rules

- Ordinary rewrite rules between solution terms:

$$m_1 \; m_2 \; \ldots \; m_k \rightarrow m_1' \; m_2' \; \ldots m_l'$$

- Rewriting takes place *only within solutions*
- Three (metaphoric) kinds of rules
  - *Heating* rules using $\rightharpoonup$ : structurally rearrange solution
  - *Cooling* rules using $\rightharpoondown$ : clean up solution after reactions
  - *Reaction* rules using $\rightarrow$ : change solution irreversibly

# CHAM Airlock

- Allows to extract molecules from encapsulated solutions
- Governed by two rules coming in a heating/cooling pair:

$$\{\!|m_1\ m_2\ \ldots\ m_k|\!\} \rightleftharpoons \{\!|m_1 \triangleleft \{\!|m_2\ \ldots\ m_k|\!\}|\!\}$$

# CHAM Molecule Configuration for IMP

- A top-level solution containing two subsolution molecules
  - One for holding the syntax
  - Another for holding the state

$$\{\!|\ \{\!|\text{Syntax}|\!\}\quad \{\!|\text{State}|\!\}\ |\!\}$$

- Example:

$$\{\!|\{\!|x := (3\,/\,(x+2))|\!\}\ \{\!|x \mapsto 1\quad y \mapsto 0|\!\}|\!\}$$

# Airlock can be Problematic

- Airlock cannot be used to encode evaluation strategies, heating/cooling rules of the form

$$\begin{array}{rcl} x := a & \rightleftharpoons & a \lhd \{\!|x := \square\,|\!\} \\ a_1 + a_2 & \rightleftharpoons & a_1 \lhd \{\!|\square + a_2|\!\} \\ a_1 + a_2 & \rightleftharpoons & a_2 \lhd \{\!|a_1 + \square\,|\!\} \end{array}$$

are problematic, because they yield ambiguity, e.g.,

$$\{\!|x := (3\,/\,(x+2))|\!\} \quad \rightleftharpoons \quad x := ((3\,/\,x) + 2)$$

$$\begin{array}{rcl} \{\!|x := (3\,/\,(x+2))|\!\} & \rightleftharpoons & \{\!|(3\,/\,(x+2)) \lhd \{\!|x := \square\,|\!\}|\!\} \\ & \rightleftharpoons & \{\!|(3\,/\,(x+2)) \quad (x := \square)|\!\} \\ & \rightleftharpoons & \{\!|(x+2) \quad (3\,/\,\square) \quad (x := \square)|\!\} \\ & \rightleftharpoons & \{\!|x \quad (\square + 2) \quad (3\,/\,\square) \quad (x := \square)|\!\} \end{array}$$

# Correct Representation of Syntax

- We need some mechanism which is not based on airlocks
- We borrow the representation approach of K
  - Term $x := (3 \,/\, (x + 2))$ represented as

$$x \curvearrowright (\square + 2) \curvearrowright (3 \,/\, \square) \curvearrowright (x := \square) \curvearrowright \square$$

- Can be achieved using heating/cooling rules of the form

$$\begin{array}{rcl}
(x := a) \curvearrowright c & \rightleftharpoons & a \curvearrowright (x := \square) \curvearrowright c \\
(a_1 \,/\, a_2) \curvearrowright c & \rightleftharpoons & a_2 \curvearrowright (a_1 \,/\, \square) \curvearrowright c \\
(a_1 + a_2) \curvearrowright c & \rightleftharpoons & a_1 \curvearrowright (\square + a_2) \curvearrowright c \\
s & \rightleftharpoons & s \curvearrowright \square
\end{array}$$

# CHAM Heating-Cooling Rules for IMP

$$
\begin{array}{rcl}
(a_1 \mathsf{+} a_2) \curvearrowright c & \rightleftharpoons & a_1 \curvearrowright (\square \mathsf{+} a_2) \curvearrowright c \\
(a_1 \mathsf{+} a_2) \curvearrowright c & \rightleftharpoons & a_2 \curvearrowright (a_1 \mathsf{+} \square) \curvearrowright c \\
(a_1 \,/\, a_2) \curvearrowright c & \rightleftharpoons & a_1 \curvearrowright (\square \,/\, a_2) \curvearrowright c \\
(a_1 \,/\, a_2) \curvearrowright c & \rightleftharpoons & a_2 \curvearrowright (a_1 \,/\, \square) \curvearrowright c \\
(a_1 \mathsf{<=} a_2) \curvearrowright c & \rightleftharpoons & a_1 \curvearrowright (\square \mathsf{<=} a_2) \curvearrowright c \\
(i_1 \mathsf{<=} a_2) \curvearrowright c & \rightleftharpoons & a_2 \curvearrowright (i_1 \,/\, \square) \curvearrowright c \\
(b_1 \,\mathsf{and}\, b_2) \curvearrowright c & \rightleftharpoons & b_1 \curvearrowright (\square \,\mathsf{and}\, b_2) \curvearrowright c \\
(\,\texttt{not}\; b) \curvearrowright c & \rightleftharpoons & b \curvearrowright (\texttt{not}\; \square) \curvearrowright c \\
(x \mathsf{:=} a) \curvearrowright c & \rightleftharpoons & a \curvearrowright (x \mathsf{:=} \square) \curvearrowright c \\
(s_1 \,;\, s_2) \curvearrowright c & \rightleftharpoons & s_1 \curvearrowright (\square \,;\, s_2) \curvearrowright c \\
s & \rightleftharpoons & s \curvearrowright \square \\
(\,\texttt{if}\; b \;\texttt{then}\; s_1 \;\texttt{else}\; s_2) \curvearrowright c & \rightleftharpoons & b \curvearrowright (\,\texttt{if}\; \square \;\texttt{then}\; s_1 \;\texttt{else}\; s_2) \curvearrowright c
\end{array}
$$

# Examples of Syntax Heating/Cooling

- The following is correct heating/cooling of syntax:

$$\{\!|x := 1\ ;\ x := (3\ /\ (x + 2))|\!\} \rightleftharpoons^* $$
$$\{\!|x := 1 \curvearrowright (\square\ ;\ x := (3\ /\ (x + 2))) \curvearrowright \square|\!\}$$

- The following is incorrect heating/cooling of syntax:

$$\{\!|x := 1\ ;\ x := (3\ /\ (x + 2))|\!\} \rightleftharpoons^* $$
$$\{\!|x := 1\ ;\ (x \curvearrowright (\square + 2) \curvearrowright (3\ /\ \square) \curvearrowright (x := \square) \curvearrowright \square)|\!\}$$

# CHAM Reaction Rules for IMP

$$
\begin{array}{rcl}
\{\!|\, x \curvearrowright c \,|\!\}\ \{\!|\, x \mapsto i \rhd \sigma \,|\!\} & \to & \{\!|\, i \curvearrowright c \,|\!\}\ \{\!|\, x \mapsto i \rhd \sigma \,|\!\} \\
(i_1 \,\texttt{+}\, i_2) \curvearrowright c & \to & (i_1 +_{Int} i_2) \curvearrowright c \\
(i_1 \,\texttt{/}\, i_2) \curvearrowright c & \to & (i_1 /_{Int} i_2) \curvearrowright c, \quad \text{when } i_2 \neq 0 \\
(i_1 \,\texttt{<=}\, i_2) \curvearrowright c & \to & (i_1 \leq_{Int} i_2) \curvearrowright c \\
(\,\texttt{true and}\ b_2) \curvearrowright c & \to & b_2 \curvearrowright c \\
(\,\texttt{false and}\ b_2) \curvearrowright c & \to & \texttt{false} \curvearrowright c \\
(\,\texttt{not true}\,) \curvearrowright c & \to & \texttt{false} \curvearrowright c \\
(\,\texttt{not false}\,) \curvearrowright c & \to & \texttt{true} \curvearrowright c \\
\{\!|\, (x \,\texttt{:=}\, i) \curvearrowright c \,|\!\}\ \{\!|\, x \mapsto j \rhd \sigma \,|\!\} & \to & \{\!|\, \texttt{skip} \curvearrowright c \,|\!\}\ \{\!|\, x \mapsto i \rhd \sigma \,|\!\} \\
(\,\texttt{skip ;}\ s_2) \curvearrowright c & \to & s_2 \curvearrowright c \\
(\,\texttt{if true then}\ s_1\ \texttt{else}\ s_2) \curvearrowright c & \to & s_1 \curvearrowright c \\
(\,\texttt{if false then}\ s_1\ \texttt{else}\ s_2) \curvearrowright c & \to & s_2 \curvearrowright c \\
(\,\texttt{while}\ b\ \texttt{do}\ s) \curvearrowright c & \to & (\,\texttt{if}\ b\ \texttt{then}\ (s\ \texttt{;}\ \texttt{while}\ b\ \texttt{do}\ s)\ \texttt{else skip}\,) \curvearrowright c \\
\texttt{vars}\ xl\ \texttt{;}\ s & \to & \{\!|\, s \,|\!\}\ \{\!|\, xl \mapsto 0 \,|\!\} \\
\\
\{\!|\, x, xl \mapsto i \,|\!\} & \rightharpoonup & \{\!|\, x \mapsto i \rhd \{\!|\, xl \mapsto i \,|\!\} \,|\!\}
\end{array}
$$

# Sample CHAM Rewriting

$$
\begin{array}{rr}
\{\!|\, \mathtt{vars}\, x, y \,;\, x := 1 \,;\, x := (3 \,/\, (x + 2)) \,|\!\} & \rightarrow \\
\{\!|\, \{\!| x := 1 \,;\, x := (3 \,/\, (x + 2)) |\!\} \quad \{\!| x, y \mapsto 0 |\!\} \,|\!\} & \rightharpoonup \\
\{\!|\, \{\!| (x := 1 \,;\, x := (3 \,/\, (x + 2))) \curvearrowright \square |\!\} \quad \{\!| x, y \mapsto 0 |\!\} \,|\!\} & \rightharpoonup^{*} \\
\{\!|\, \{\!| (x := 1 \,;\, x := (3 \,/\, (x + 2))) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 0 \quad y \mapsto 0 |\!\} \,|\!\} & \rightharpoonup \\
\{\!|\, \{\!| (x := 1) \curvearrowright (\square \,;\, x := (3 \,/\, (x + 2))) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 0 \quad y \mapsto 0 |\!\} \,|\!\} & \rightharpoonup \\
\{\!|\, \{\!| (x := 1) \curvearrowright (\square \,;\, x := (3 \,/\, (x + 2))) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 0 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightarrow \\
\{\!|\, \{\!| \mathtt{skip} \curvearrowright (\square \,;\, x := (3 \,/\, (x + 2))) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightharpoondown \\
\{\!|\, \{\!| (\mathtt{skip} \,;\, x := (3 \,/\, (x + 2))) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightharpoondown \\
\{\!|\, \{\!| (x := (3 \,/\, (x + 2))) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightharpoonup^{*} \\
\{\!|\, \{\!| x \curvearrowright (\square + 2) \curvearrowright (3 \,/\, \square) \curvearrowright (x := \square) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightarrow \\
\{\!|\, \{\!| 1 \curvearrowright (\square + 2) \curvearrowright (3 \,/\, \square) \curvearrowright (x := \square) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightharpoondown \\
\{\!|\, \{\!| (1 + 2) \curvearrowright (3 \,/\, \square) \curvearrowright (x := \square) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightarrow \\
\{\!|\, \{\!| 3 \curvearrowright (3 \,/\, \square) \curvearrowright (x := \square) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightarrow^{*} \\
\{\!|\, \{\!| (x := 1) \curvearrowright \square |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightarrow \\
\{\!|\, \{\!| \mathtt{skip} \curvearrowright \square |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} & \rightarrow \\
\{\!|\, \{\!| \mathtt{skip} |\!\} \quad \{\!| x \mapsto 1 \rhd \{\!| y \mapsto 0 |\!\} |\!\} \,|\!\} &
\end{array}
$$

# CHAM in Rewriting Logic

- CHAM rules cannot be used unchanged as rewrite rules
  - They need to only apply in solutions, not anywhere they match
- We represent each CHAM rule

$$m_1 \ m_2 \ \ldots \ m_k \to m'_1 \ m'_2 \ \ldots \ m'_l$$

into a rewrite logic rule

$$\{\!|m_1 \ m_2 \ \ldots \ m_k \ ms|\!\} \to \{\!|m'_1 \ m'_2 \ \ldots \ m'_l \ ms|\!\}$$

# CHAM of IMP in Maude

- See files in **`imp-cham.zip`**